

---

# The COMPUTATIONAL ENGINEERING SCIENCES

---

**A. J. Baker**

*Engineering Science Graduate Program  
The University of Tennessee, Knoxville*

## TABLE OF CONTENTS

<b>1</b>	<b>CES.</b>	<b>COMPUTATIONAL ENGINEERING SCIENCE</b>	<b>1</b>
	FE.1	Engineering simulation	1
	FE.2	A problem solving environment	2
	FE.3	Problem statements in engineering	4
	FE.4	Decisions on forming $WS^N$	6
	FE.5	Discrete approximate $WS^h$ implementation	8
	FE.6	Summary	9
	FE.7	References	10
<b>2</b>	<b>PS.</b>	<b>PROBLEM STATEMENTS</b>	<b>11</b>
	PS.1	Engineering simulation	11
	PS.2	Continuum mechanics viewpoint	12
	PS.3	Continuum conservation law forms	10
	PS.4	Constitutive closure for conservation law PDEs	14
	PS.5	Continuum mechanics problem statements	18
	PS.6	References	19
<b>3</b>	<b>IM.</b>	<b>SOME INTRODUCTORY MATERIAL</b>	<b>21</b>
	IM.1	Introduction	21
	IM.2	Multi-dimensional PDEs, separation of variables	23
	IM.3	Theoretical foundations, GWS	27
	IM.4	Legacy FD connections to WS	28
	IM.5	An FD approximate solution	30

	IM.6	Lagrange interpolation polynomials	32
	IM.7	Summary, FE trial space basis	33
	IM.8	Problems	34
	IM.9	References	34
<b>4</b>	<b>HC.</b>	<b>HEAT CONDUCTION</b>	<b>35</b>
	HC.1	A heat conduction example	35
	HC.2	The approximation, minimization of error	37
	HC.3	$GWS^N$ discrete implementation, FE basis	38
	HC.4	Finite element matrix library	41
	HC.5	Assembly of $\{WS\}_e$ to form $GWS^h$	43
	HC.6	Solution accuracy, error distribution	45
	HC.7	Boundary heat flux computation	46
	HC.8	Problems	47
<b>5</b>	<b>HT1.</b>	<b>STEADY HEAT TRANSFER, <math>n = 1</math></b>	<b>49</b>
	HT1.1	Introduction	49
	HT1.2	Steady heat transfer on $n = 1$	50
	HT1.3	Linear FE basis matrix library	52
	HT1.4	Object-oriented $GWS^h$ programming	55
	HT1.5	Higher degree polynomial FE bases	58
	HT1.6	Theoretical asymptotic error estimate	62
	HT1.7	Handling source data	63
	HT1.8	Temperature dependent conductivity, non-linearity	68
	HT1.9	Static condensation, $p$ -elements	72
	HT1.10	Summary	74
	HT1.11	Problem assignments	76
	HT1.12	Computer exercises	76
	HT1.13	References	77
<b>6</b>	<b>CM1</b>	<b>COMPUTATIONAL MECHANICS, <math>n = 1</math></b>	<b>79</b>
	CM1.1	Introduction	79
	CM1.2	The Euler-Bernoulli beam	80
	CM1.3	Euler-Bernoulli beam $GWS^h$ reformulated	85
	CM1.4	The Timoshenko beam	92
	CM1.5	Mechanical vibration of a beam	99
	CM1.6	Aerodynamics, potential flow	106
	CM1.7	Electromagnetic plane wave propagation	111
	CM1.8	Heat transfer from a finned cylinder	112

	CM1.9 Problems	122
	CM1.10 Computer labs	122
	CM1.11 References	124
<b>7</b>	<b>HTn. STEADY HEAT TRANSFER, <math>n &gt; 1</math></b>	<b>125</b>
	HTn.1 Introduction	125
	HTn.2 Multi-dimensional FE bases and elements	126
	HTn.3 Multi-dimensional FE operations for $\{N_k(\xi)\}$	129
	HTn.4 The $k = 1$ natural coordinate FE matrix library	132
	HTn.5 $\{WS\}_e$ implementation via $\{N_1\}$ , $n = 2$	137
	HTn.6 The tensor product element family	139
	HTn.7 Gauss numerical quadrature	141
	HTn.8 The GWS <sup>h</sup> template essence for $n$ -D heat transfer	146
	HTn.9 Accuracy, convergence revisited	149
	HTn.10 Summary	150
	HTn.11 Problems	151
	HTn.12 Computer exercises	152
	HTn.13 References	152
<b>8</b>	<b>FDV. FINITE DIFFERENCES OF OPINION</b>	<b>153</b>
	FDV.1 The FD-FE correlation	153
	FDV.2 The FV-FE correlation	156
	FDV.3 Summary	161
	FDV.4 Problems	161
	FDV.5 Reference	161
<b>9</b>	<b>CD1. CONVECTION-DIFFUSION, <math>n=1</math></b>	<b>163</b>
	CD1.1 Introduction	163
	CD1.2 Galerkin weak statement	165
	CD1.3 GWS <sup>h</sup> completion for time-dependence	166
	CD1.4 GWS <sup>h</sup> + $\theta$ TTS template	167
	CD1.5 Asymptotic error estimates	170
	CD1.6 Verification test cases	171
	CD1.7 Dispersive error characterization	175
	CD1.8 The Taylor weak statement	177
	CD1.9 Verification problem statements revisited	181
	CD1.10 Unsteady heat conduction	182
	CD1.11 Summary	185

	CD1.12 Problem assignments	185
	CD1.13 Computer labs	186
	CD1.14 References	186
<b>10</b>	<b>CD<math>n</math>. CONVECTION-DIFFUSION, <math>n&gt;1</math></b>	<b>187</b>
	CD $n$ .1 The problem statement	187
	CD $n$ .2 GWS <sup><i>h</i></sup> essential formulation	188
	CD $n$ .3 Matrix library additions and templates	190
	CD $n$ .4 Accuracy, convergence, stability	192
	CD $n$ .5 Linear algebra, matrix iteration	195
	CD $n$ .6 Newton and AF TP quasi-Newton jacobians	200
	CD $n$ .7 Verification, benchmarking and validation	202
	CD $n$ .8 Mass transport, the rotating cone	203
	CD $n$ .9 The gaussian plume	206
	CD $n$ .10 The steady $n$ -D Peclet problem	208
	CD $n$ .11 An $n = 3$ validation experiment	210
	CD $n$ .12 Summary	214
	CD $n$ .13 Problems	215
	CD $n$ .14 Computer exercises	215
	CD $n$ .15 References	216
<b>11</b>	<b>CM<math>n</math>. COMPUTATIONAL MECHANICS, <math>n&gt;1</math></b>	<b>217</b>
	CM $n$ .1 Introduction	217
	CM $n$ .2 Structural mechanics	218
	CM $n$ .3 Virtual work FE implementation	221
	CM $n$ .4 Plane stress/strain, $\{N_1(\zeta)\}$ implementation	224
	CM $n$ .5 A plane stress computer lab	229
	CM $n$ .6 Fluid mechanics, buoyant flow	234
	CM $n$ .7 GWS <sup><i>h</i></sup> + $\theta$ TS algorithm, FE implementation	237
	CM $n$ .8 An isothermal INS benchmark	239
	CM $n$ .9 Convection heat transfer experiment	242
	CM $n$ .10 Mechanical vibrations, normal mode GWS <sup><i>h</i></sup>	249
	CM $n$ .11 Vibrations computer experiment	251
	CM $n$ .12 Problems	257
	CM $n$ .13 Computer exercises	258
	CM $n$ .14 References	259
	Index	261

Enclosure DVD